Calculation of Separation Points in Incompressible Turbulent Flows

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The purpose of this paper is to evaluate the accuracy with which the location of turbulent separation can be predicted on two-dimensional and axisymmetric bodies. The evaluation was made by studying a considerable number of flows that had separation. Calculated separation points were compared with the experimentally measured location. Four methods of predicting separation in turbulent flow were evaluated. They were Goldschmied's method, Stratford's method, Head's method, and the Cebeci-Smith method. It was concluded from the study that the last three listed methods predict separation points with the reliability and accuracy needed for aerodynamic design purposes.

Nomenclature

c = chord
c_f = local skin friction coefficient, $\tau_w/(1/2)\rho u_e^2$
C_p = pressure coefficient, $(p - p_m)/(1/2\rho u_m^2)$
D = diameter
h = total head
H = shape factor, δ^*/θ
k = mixing-length constant
L = reference body length
p = pressure
R_c = chord-Reynolds number, $u_\infty c/\nu$
R_D = diameter-Reynolds number, $u_\infty D/\nu$
R_L = length-Reynolds number, $u_\infty L/\nu$
R_x = local Reynolds number, $u_e x/\nu$
R_θ = Reynolds number, $u_e\theta/\nu$
u,v = x and y components of velocity, respectively
u^* = friction velocity, $(\tau_w/\rho)^{1/2}$
x = streamwise distance
y = distance normal to the surface of the body
α = angle of attack
δ = boundary-layer thickness
δ^* = displacement thickness, $\int_0^\infty (1 - u/u_e)dy$
θ = momentum thickness, $\int_0^\infty u/u_e(1 - u/u_e)dy$
μ = dynamic viscosity
ν = kinematic viscosity
ρ = density
τ = shear stress
ϕ = angle from stagnation point

Subscripts

e = edge of the boundary layer
m = minimum pressure point
sep = separation point
tr = transition
w = wall
∞ = freestream conditions

Introduction

IN many problems, it is necessary to know whether the boundary layer (either laminar or turbulent) will separate from the surface of a specific body. If it does, it is also necessary to know accurately where the flow separation will occur. This is quite important in many design problems. In the design of airfoils or hydrofoils, it is necessary to avoid flow separation in order to keep drag levels low. In designing for high lift, predicting separation points is a crucial part of the design problem.

For steady two-dimensional and axisymmetric flows, the separation point is defined as the point where the wall shear stress τ_w is equal to zero, i.e.,

$$(\partial u/\partial y)_w = 0 \qquad (1)$$

With high-speed computers, the governing boundary-layer equations for laminar flow can be solved exactly, and consequently, the laminar separation point can be determined almost exactly. In addition, there are several simple methods which do not require the solution of the boundary-layer equations in their differential form, and can be used to predict separation points quite satisfactorily. The momentum integral method of Thwaites and the method of Stratford are examples of two such methods. The latter does not even require the solution of the laminar boundary-layer equations. For a given pressure distribution [$C_p(x)$, for example], the expression

$$C_p^{1/2}(x\, dC_p/dx) \qquad (2)$$

is calculated around the body. Laminar separation is predicted when it reaches a value of 0.102.

The prediction of the separation point in turbulent flows, on the other hand, is a much more difficult job. As a result of the presence of the time mean of the fluctuating quantities appearing in the governing equations, an exact solution of the boundary-layer equations for turbulent flows is impossible. Consequently, when the equations are solved with some suitable assumption for these quantities, the solutions contain empiricism, and must be checked against experiment.

The current prediction methods on the subject can be divided into two groups. In one group are methods that require the detailed solution of the boundary-layer equations. These methods are either of differential type (meaning that partial-differential equations are solved) or of integral type (meaning that momentum integral or energy integral equations are solved). Reference 1 presents a critical evaluation of these methods for two-dimensional incompressible turbulent flows. In differential methods, the parameter used to predict the separation point is the zero wall shear stress. In integral methods, on the other hand, the shape factor $H = \delta^*/\theta$ is usually used to locate the separation point. In integral methods, as the flow approaches separation, the value of H increases. *Separation of the flow is assumed to occur when H reaches a value between* 1.8 *and* 2.4. In some cases,§ however, the value of H increases rapidly near separation, and then begins to decrease. Then, the point corresponding to the maximum value of H is taken as the separation point.

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§ These cases correspond to flows, for which the calculations are made using an experimental pressure distribution.

In another group are methods that do not require detailed boundary-layer calculations. Separation is predicted by simple formulas, or by simple differential equations that are very fast and easy to apply. These methods also utilize the composite nature of the turbulent boundary layer. For example, Stratford[2] divides the turbulent boundary layer into inner and outer regions and bases his analysis on two assumptions: 1) in the outer region, the pressure forces cause a direct reduction in dynamic head and 2) in the inner region, the pressure force is balanced by the shear-force gradient. Goldschmied's method also treats the boundary layer consisting of inner and outer regions. His analysis is based on the assumptions of inner-region similarity under any pressure gradient, and of a constant total-head line at a fixed distance from the wall.

In this paper, we report the accuracy of several current methods for predicting the turbulent boundary-layer separation point. In particular, we consider a differential method (the Cebeci-Smith or CS method), a momentum integral method (Head's method[3]), and two simple methods—namely, the methods of Stratford[2] and Goldschmied.[4] The differential method of Cebeci and Smith is discussed in Refs. 5 and 6. For this reason, only the other three methods are discussed briefly in the next section.

Methods for Predicting Turbulent Boundary-Layer Separation

Head's Method

Head's method is an integral method that can be used both for calculating the boundary-layer parameters, as well as for predicting the position of separation in turbulent flows. It uses the momentum integral equation

$$d\theta/dx + (H+2)(\theta/u_e)(du_e/dx) = c_f/2 \tag{3}$$

and two auxiliary equations—namely, the Ludwieg-Tillman expression for the skin-friction coefficient

$$c_f = 0.246(10)^{-0.678H} R_\theta^{-0.268} \tag{4}$$

and a shape factor $G(H)$ relationship obtained from the entrainment properties of the turbulent boundary layer. The latter is also related to another shape factor H_1. The entrainment and the shape factor relationships are as follows.

Entrainment Relation

$$(1/u_e)(d/dx)(u_e\theta H_1) = 0.0299(H_1 - 3.0)^{-0.6169} \tag{5}$$

Shape Factor Relation

$H_1 = G(H)$ where

$$G(H) = \begin{cases} 0.8234(H-1.1)^{-1.287} & H \le 1.6 \\ 1.5501(H-0.6778)^{-3.064} + 3.3 & H \ge 1.6 \end{cases} \tag{6}$$

This method, like most integral methods, uses the shape factor H as the criterion for separation. Although it is not possible to give an exact value of H corresponding to separation, when H is between 1.8 and 2.4, separation is assumed to exist. The difference between the lower and upper limits of H makes very little difference in locating the separation point, since close to separation the shape factor increases quickly.

Stratford's Method

Stratford's method for turbulent flows is a simple one which uses only the pressure distribution to predict boundary-layer separation. It does not require detailed boundary-layer calculations like the methods of Refs. 3 or 6. Presently, there are several methods based on the ideas set forth in this method.[9,10] However, the accuracy of these methods is similar to that of Stratford's, and so the methods are not considered in detail in this report.

Stratford's method is based upon the idea of dividing the boundary layer into outer and inner portions. It follows the principles successfully adopted for laminar flows. According to this method, separation for turbulent boundary layers is predicted from the following expression

$$F(x) \equiv C_p(x\, dC_p/dx)^{1/2}(10^{-6}R_x)^{-1/10} = 1.25\,k \tag{7}$$

The above expression applies for an adverse pressure gradient flow starting from the leading edge, as well as fully turbulent flow everywhere. When there is a region of laminar flow, or a region of turbulent flow with a favorable pressure gradient, Stratford makes the assumption that at the minimum pressure point $x = x_m$ the velocity profile is approximately that of a flat-plate turbulent boundary layer starting from a false origin $x = x'$. In this case, we replace x by $(x - x')$ in (7), and take the value of R_x as $u_m(x_m - x')/\nu$. Then $x_m - x'$ is given by[5]

$$x_m - x' = 58\frac{\nu}{u_m}\left[\frac{u_{tr}}{\nu}\int_0^{x_{tr}}\left(\frac{u_e}{u_m}\right)^5 dx\right]^{3/5} + \int_{x_{tr}}^{x_m}\left(\frac{u_e}{u_m}\right)^4 dx \tag{8}$$

With the expression given by Eq. (8), Eq. (7) can be used to predict the separation point in turbulent flows. In order to do this, however, it is necessary to assume a value for k, which, according to the mixing length theory, is 0.4. This means that the right-hand side of Eq. (7) should be 0.5, but a comparison with experiment, according to Stratford, suggests a smaller value of $F(x)$ around 0.35 and 0.40. For a typical turbulent boundary-layer flow with an adverse pressure gradient, it is found that $F(x)$ increases as separation is approached, and decreases after separation. For this reason, after applying his method to several flows with turbulent separation, Stratford observed that if the maximum value of $F(x)$ a) is greater than 0.40, separation is predicted when $F(x) = 0.40$; b) lies between 0.35 and 0.40, separation occurs at the maximum value; c) is less than 0.35, then separation does not occur.

Goldschmied's Method

Goldschmied's separation criterion,[4] like Stratford's method, is based on the existence of inner and outer regions in the turbulent boundary layer. Goldschmied assumes that there is a line in the inner region at a constant distance y_c from the wall, with constant total head h_c, such that

$$h_c = p + \tfrac{1}{2}\rho u_c^2 \tag{9}$$

Furthermore, since the line is in a region where the law of the wall applies, he assumes it to be independent of pressure distribution, and selects the outer edge of the inner region at the start of the adverse pressure gradient as the starting point of the line. He assumes that the outer edge of the inner region is characterized approximately by $u/u^* = 20$ and $yu^*/\nu = 500$. Then the total head at the start of adverse pressure gradient can be written as

$$h_m = p_m + \tfrac{1}{2}(20u_m^*)^2 \tag{10}$$

Then from Eqs. (9) and (10),

$$p_m - p + \tfrac{1}{2}\rho[400(\tau_w/\rho)] = \tfrac{1}{2}\rho u_c^2 \tag{11}$$

since $u_m^* = (\tau_w/\rho)^{1/2}$. Dividing both sides of Eq. (11) by u_m^2 and rearranging gives

$$(u_c/u_m)^2 = (p_m - p)/\tfrac{1}{2}\rho u_m^2 + 400(\tau_w/\rho u_m^2) \tag{12}$$

If the following terms are defined,

$$c_{f_m} = \tau_w/\tfrac{1}{2}\rho u_m^2 \quad \text{and} \quad C_p = (p - p_m)/\tfrac{1}{2}\rho u_m^2$$

Eq. (12) becomes

$$u_c/u_m = (200c_{f_m} - C_p)^{1/2} \tag{13}$$

Making use of the laminar sublayer and the law of the wall, he shows further that at separation, the expression $u_c/u_m = 1/3.45[c_{f_m}/2]^{1/2}$ is so small that it can be neglected. Then Eq. (13) reduces to

$$C_{p\text{sep}} = 200c_{f_m} \tag{14}$$

and becomes the separation criterion for Goldschmied's method.

Comments on the Four Methods

In the previous section, we presented a brief description of the fluid mechanics aspects of the four methods. Of these, Goldschmied's method is the simplest one to use. This method takes into account some of the earlier history of the boundary layer, since c_{f_m} depends on the flow history. After the minimum pressure point is passed, details of the flow are ignored. Consider Fig. 1. Up to the minimum pressure point (m) a general accelerating flow is assumed as sketched. Then Goldschmied's method predicts separation when C_p reaches a certain level as indicated, entirely independent of path; the rise may be slow or fast.

Stratford's method predicts separation when $C_p(xdC_p/dx)^{1/2}$ reaches a certain value. The method takes into account C_p, x and dC_p/dx, so that path and distance now are considered to a certain extent. But Stratford's method will give the same answer for a number of pressure rise paths. Assume point s is the separation point for path a according to Stratford's method. Then x, C_p, and dC_p/dx are fixed. But paths b and c start from the same point and end at the same point with the same set of terminal values. It is easy to show that certain paths b and c do not exceed Stratford's separation criterion at intermediate values of x. Hence we have shown that Stratford's method does not take into account all the details of a pressure rise.

Head's method, being a differential equation solved as a function of pressure distribution, can distinguish between the pressure distributions a, b, and c of Fig. 1. The difficulty is that it still has considerable approximations in it, being a momentum integral type of equation. One case where it fails is in flow of equilibrium boundary layers. This kind of method will eventually predict separation where, in fact, separation does not occur.

The partial differential equation method such as the CS method also responds to full details of the pressure history. Furthermore, it predicts equilibrium flows correctly, although that problem is more difficult than predicting nonequilibrium flows.

In summary, then, the four methods have the following features:

1) Goldschmied: only sets a separation C_p level, and takes no account of the shape of the pressure distribution. Accuracy of the results is vitally dependent on the precision of estimating c_{f_m}. This method is not applicable to axisymmetric flows.
2) Stratford: takes partial account of the shape of the pressure rise curve, but will give the same answers for a variety of pressure distributions. This method is not applicable to axisymmetric flows.
3) Head: takes complete account of the shape of the pressure distribution, but uses a momentum inetgral equation with its approximations. This method is not applicable to axisymmetric flows.
4) Cebeci-Smith: takes complete account of the shape of the pressure distribution by use of more exact differential equations than Head's. This method is applicable to axisymmetric flows.

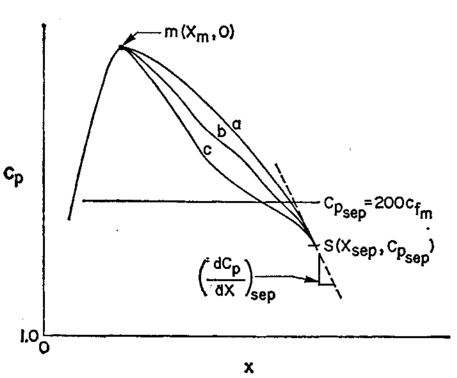

Fig. 1 Schematic for comparison of separation criteria.

The Problem of Predicting Separation from Experimental Data

The general methods of calculation have been described, and some of their basic features have just been summarized. But still another problem needs discussion—the use of experimental pressure distributions. To evaluate the accuracy of predicting separation points, we must examine experimental data where the flows do separate. Otherwise, there is no base for assessment. Typical separating flows have a peculiar pressure distribution function. The pressure distribution flattens out because of the separated region. The effect is shown in Figs. 2, 5, 10, and 11, among others. After a short transition region, the pressure becomes essentially constant. In performing boundary-layer calculations, this is perceived as a flat-plate flow. Therefore, boundary-layer methods may or may not predict separation, depending on whether they have an optimistic or a conservative basis.

Examination of the region of transition between the variable pressure region and the constant pressure separation region shows that it is short. See, for example, $\alpha = 12°$, Fig. 11. The boundary-layer equations legitimately apply to some place within the transition region. But beyond this point the equations do not properly apply, and furthermore, separation is not likely to be predicted. To avoid this dilemma, we and others attacking this problem simply extrapolate the pressure distribution following the guidelines given by inviscid theory. Extrapolation is done graphically, but errors should not be great, because the transition region is generally short. The flow is so near separation by the time the extrapolation is commenced that any reasonable extrapolation will give nearly the same location of separation. The extrapolation would have to appear absurd before significant changes would occur.

Then, given this extrapolated pressure distribution, what do we find? If separation is clearly predicted before the start of the transition region, we have a poor prediction, because obviously the flow did not separate in that region.

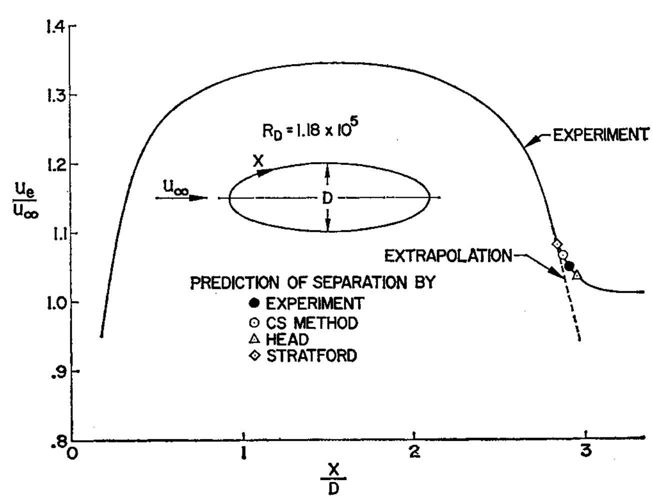

Fig. 2 Separation points for Schubauer's elliptic cylinder.

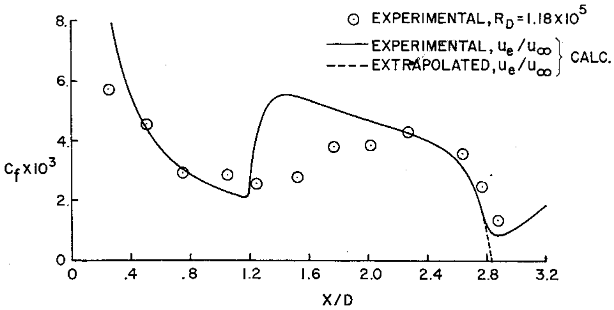

Fig. 3 Calculated and experimental local skin-friction coefficients for Schubauer's elliptic cylinder. The calculations were made by the CS boundary-layer method.

Predictions made using the arbitrary extrapolation of pressure distribution are found to agree well with experiment. Furthermore, for the CS method, the minimum in c_f occurs at about the same point. A typical extrapolated pressure distribution and the accompanying c_f calculations are shown in Figs. 2 and 3. It is seen that the c_f obtained from following the experimental pressure distribution has its minimum at very nearly the same place where $c_f = 0$ for the extrapolated case. A large number of studies of this sort showed good agreement. Hence either method is acceptable: to look for the minimum value of c_f using the measured pressure distribution, or to look for the point where $c_f = 0$, using an extrapolated pressure distribution.

The same problem occurs regardless of the method that is used, and the treatment in terms of H, etc., is similar. The problem of relaxing pressure does not normally occur for inviscid flows. In any case, it is assumed that all the knowledge available is used. It will be shown now by a number of examples that separation points for arbitrary experimental flows can be predicted well. That being so, we can state that separation points on theoretical inviscid flows can be predicted as well.

Comparison of Calculated and Experimental Separation Points

In this section, we will consider several experimental pressure distributions which include observed or measured boundary-layer separation, and apply the four separation-prediction methods discussed previously to these pressure distributions. It is important to note that near separation, the behavior of these methods with an experimental pressure distribution is quite different from that with an inviscid pressure distribution. The pressure distribution near the point of separation may be a characteristic of the phenomenon of separation, and inclusion of it in the specification of the flow is equivalent to being told the position of separation.[9] For this reason, use of these separation-prediction methods with an experimental pressure distribution will only show their behavior close to separation, and indicate whether the theoretical assumptions used in these methods are self-consistent. When one considers an experimental pressure distribution with separation and uses the CS method, it is quite possible that the wall shear stress at the experimental separation point may not reach zero. It may decrease as the separation is approached, and may start to increase past the separation point. Similarly, the shape factor H in Head's method may not show a continuous increase to the position of separation. Depending on the pressure distribution which is distorted by the separation flow, the shape factor may even start to decrease after an increase. All that can be learned from a study such as the one conducted here is how these methods behave close to separation, and whether they predict an early separation or no separation at all.

While there is much good data available for comparing calculated separation points with experimental separation points for two-dimensional bodies, there is not much good data for axisymmetric bodies.

In this study, we have tested the previously-discussed separation prediction methods for a large number of two-dimensional flows. However, only a few cases are considered for axisymmetric flows. During the study, it became necessary to make certain assumptions in applying Goldschmied's method. According to this method, it is necessary to calculate the local turbulent skin-friction coefficient at the minimum pressure point. In the cases studied here, however, the flow is generally laminar at the minimum pressure point and becomes turbulent downstream of that point. In these cases, the calculated local skin-friction coefficient for turbulent flow was extrapolated to the minimum pressure point.

It was also observed that Stratford's method has better agreement with experiment if the range of $F(x)$ was slightly changed from that given in a previous section—namely, if the maximum value of $F(x)$ a) is greater than 0.50, separation is predicted when $F(x) = 0.50$; b) lies between 0.30 and 0.40, separation occurs at the maximum value; c) is less than 0.30, then separation does not occur.

Results are reported by marking calculated separation points on the pressure distribution curves because, in some cases, experimental separation points could be inferred only from the pressure distributions. This method of presentation, therefore, helps the reader in assessing the accuracy of the various theoretical approaches studied.

Results for Schubauer's Elliptic Cylinder

Figures 2 and 3 show the results for Schubauer's elliptic cylinder,[11] which has a 3.98-in. minor axis D. The experimental pressure distribution was given at a freestream velocity of $u_\infty = 60$ ft/sec, corresponding to a Reynolds number of $R_D = 1.18 \times 10^5$. The extent of the transition region was between $x/D = 1.25$ and $x/D = 2.27$, and experimental separation was indicated at $x/D = 2.91$.

In the calculations, the transition point was assumed at $x/D = 1.25$. Figure 2 shows the results. It is interesting to note that while three methods predicted separation, the fourth method (Goldschmied's), did not predict any separation.

Figure 3 shows a comparison of calculated and experimental local skinfriction values. The calculations were made by using the CS method. When the experimental pressure distribution was used, the local skin-friction coefficient began to increase near separation because of the pressure distribution which was distorted by the separating flow. However, when the calculations were repeated by using an extrapolated velocity distribution which could be obtained by an inviscid method, the skin friction went to zero at $x/D = 2.82$.

Figures 2 and 3 are convenient to illustrate the difference in treatment between experimental and theoretical pressure distribution data. As shown in Fig. 2, because of separation the adverse pressure gradient becomes less severe and approaches zero. If a boundary-layer method predicted separation, somewhat late separation would not be predicted at all, and gradually the method would converge toward flat-plate results because of the final constancy of the pressure. Figure 3 illustrates the effect. Separation is not really predicted, but there is a clear and well-defined minimum in skin friction. If the experimental pressure distribution is extrapolated following potential theory as in Fig. 2, then separation is predicted by the CS method as in Fig. 3. It is seen that the minimum c_f point from experimental data, and the $c_f = 0$ point from the inviscid extrapolation, agree well. Therefore, establishment of accuracy of methods by application to experimental pressure distributions seems justified.

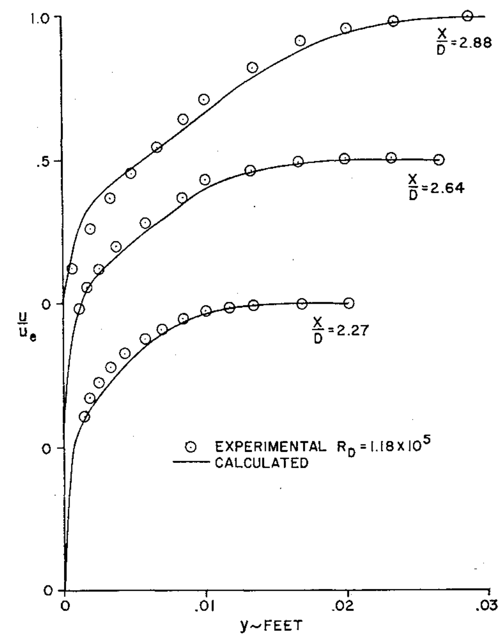

Fig. 4 Calculated and experimental velocity profiles for Schubauer's elliptic cylinder. The calculations were made by the CS boundary-layer method.

Figure 4 shows a comparison of calculated and experimental velocity profiles at various x/D locations for the same body. In general, the agreement for both laminar and turbulent boundary layers seems to be quite satisfactory.

Results of Roshko's Circular Cylinder

Figure 5 shows the predicted separation points, together with the experimental points for Roshko's circular cylinder,[12] for two diameter Reynolds numbers $R_D = 6.7 \times 10^5$ and 8.4×10^6 that are within the so-called "supercritical" and "transcritical" Reynolds number ranges.

According to Roshko, at $R_D = 6.7 \times 10^5$ a separation bubble exists for angles 100–120°. This can be inferred from the pressure distribution. However, it is difficult to find the exact location of the reattachment point. Also, the turbulent separation point in this case must be very close to the reattachment point. Thus, the extent of attached turbulent flow is probably very small, possibly 115–120°.

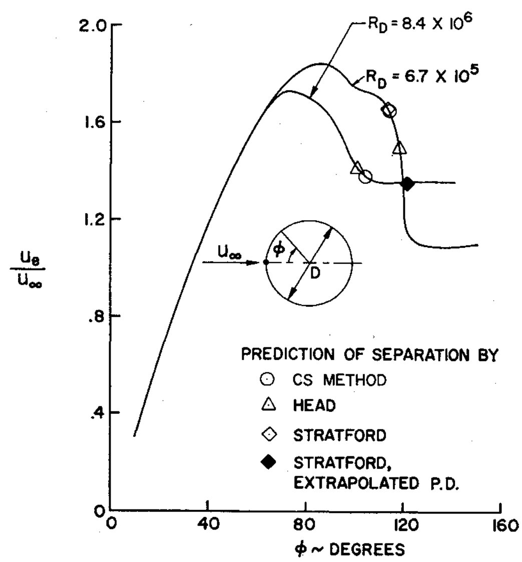

Fig. 5 Separation points for Roshko's circular cylinder.

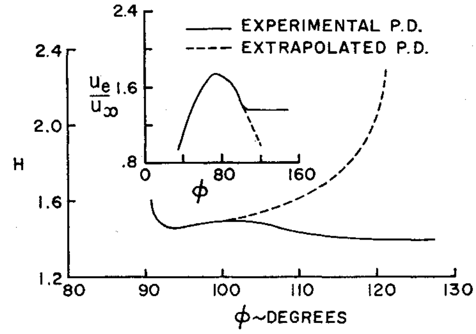

Fig. 6 Variation of shape factor with two pressure distributions for Roshko's circular cylinder. Calculations were made by Head's method for $R_D = 8.4 \times 10^6$.

At higher Reynolds number, $R_D = 8.4 \times 10^6$, on the other hand, the laminar separation region is much smaller and the extent of the turbulent flow region is fairly large, as evident from the forward movement of the minimum pressure point and the smaller pressure peak.

For both Reynolds numbers, Goldschmied's method did not predict separation. On the other hand, in both cases Head's method and CS method predicted separation. For $R_D = 6.7 \times 10^5$, Stratford's method predicted separation and for $R_D = 8.4 \times 10^6$ it did not. In the latter case, $F(x)$ was less than 0.2. However, when the velocity distribution was extrapolated (see Fig. 6), then separation was predicted.

Figure 6 shows the variation of shape factor for the experimental and extrapolated velocity distributions at $R_D = 8.4 \times 10^6$. The calculations were made by Head's method. As expected with the extrapolated velocity distribution (which is similar to inviscid velocity distribution), the shape factor quickly increases close to separation. On the other hand, with the experimental velocity distribution, the shape factor reaches a maximum and then starts to decrease.

Results for Several Airfoils

Figures 7–12 show the results obtained for several airfoils where separation was observed. The results for the pressure distribution of Schubauer and Klebanoff[13] are shown in Fig. 7. This pressure distribution was observed over an airfoil-like body at a Reynolds number per foot of 0.82×10^6. The experimental separation point was given at 25.7 ± 0.2 ft. The predictions of all methods are quite good.

As shown in Fig. 8, agreement between the CS method and experiment is also very good for Newman's airfoil.[7] On the other hand, the other methods predict a slightly early separation.

For the pressure distribution of Figs. 9–12, the experimental separation points were not given, but can be inferred from the shape of the pressure distribution. The results show that, except at very high angles of attack, both boundary-layer methods predict separation at approximately the same streamwise locations, and generally close to the characteristic "flattening" in the pressure distribution curves. Stratford's method predicts a slightly earlier separation than that given

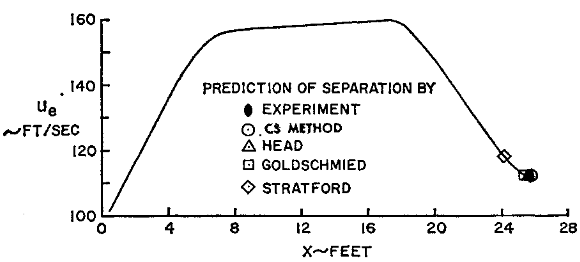

Fig. 7 Separation points for the airfoil-like body of Schubauer and Klebanoff.

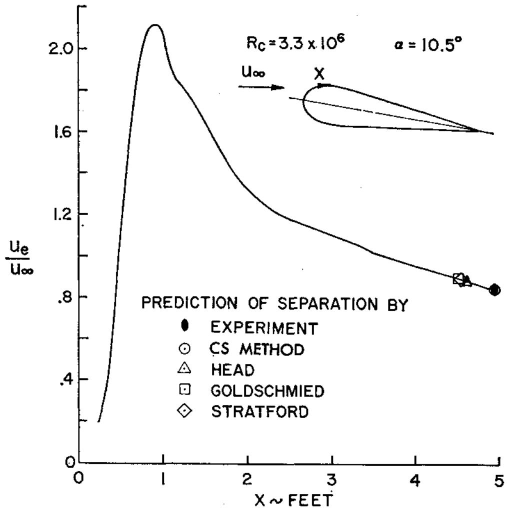

Fig. 8 Separation points for Newman's airfoil.

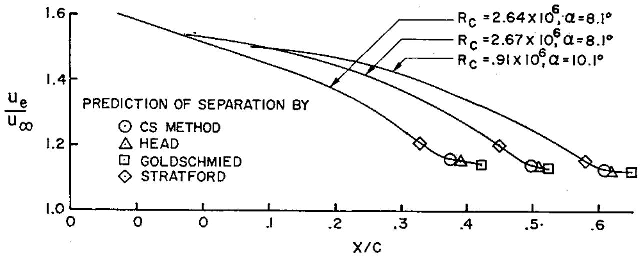

Fig. 9 Predicted separation points for the experimental pressure distribution on the NACA 65(216)-222 airfoil.

by the boundary-layer methods. On the other hand, Goldschmied's method shows results that are somewhat inconclusive, predicting early separation in some cases and late separation in others.

Results for Axisymmetric Flows

For axisymmetric flows, Head's, Stratford's, and Goldschmied's methods cannot be used to predict the position of separation in their present form. For this reason, only

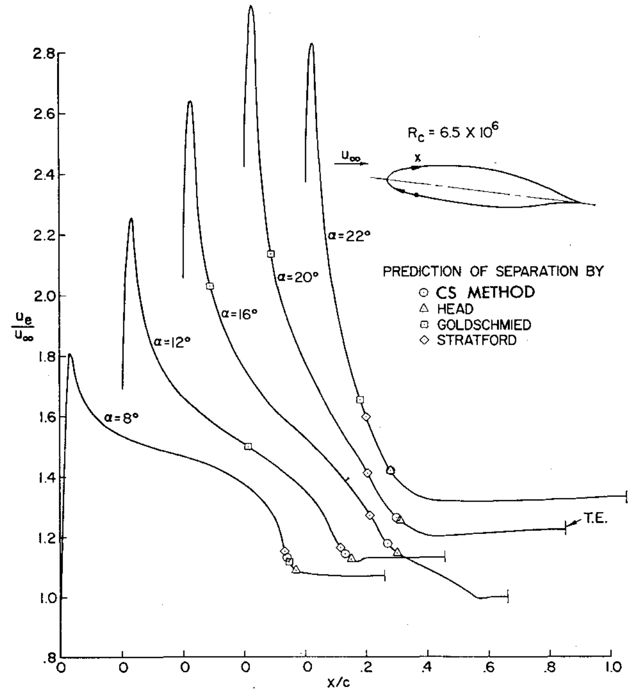

Fig. 11 Predicted separation points for the experimental pressure distribution on the NACA 66, 2-420 airfoil.

the CS method was used to predict the separation points in such flows.

Table 1 shows the results for the Murphy bodies.[18] The experimental separation points were obtained accurately by the "dust" technique. The calculated separation points

Table 1 Comparison of calculated and experimental separation points for the bodies of revolution of Murphy[a]

		x_{sep} (in.)	
Tail shape	$R_L \times 10^{-6}$	Experiment	CS method
A-2	6.0	59.1	59.4
C-2	6.0	58.3	58.2
C-4	6.6	No separation	No separation

[a] See Ref. 18.

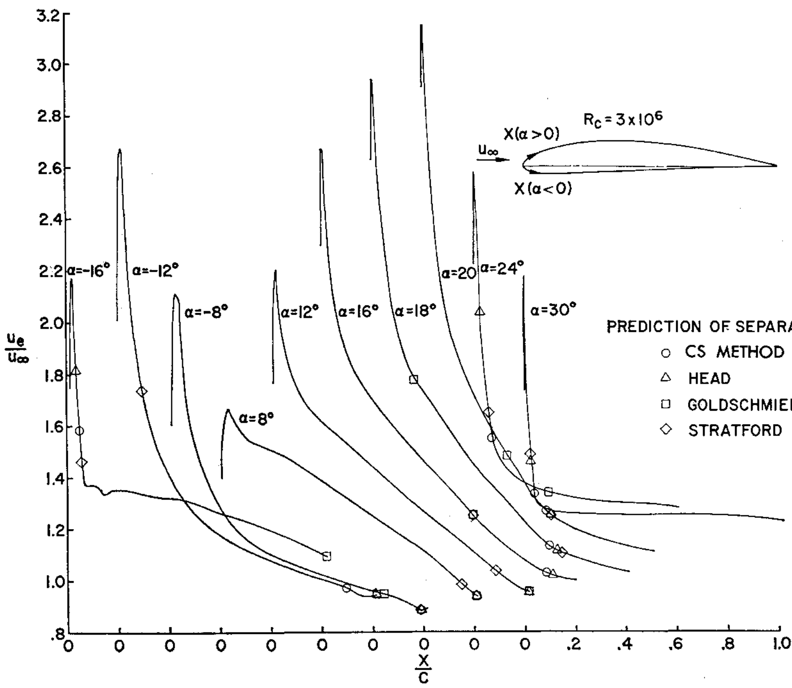

Fig. 10 Predicted separation points for the experimental pressure distribution on the NACA 4412 airfoil.

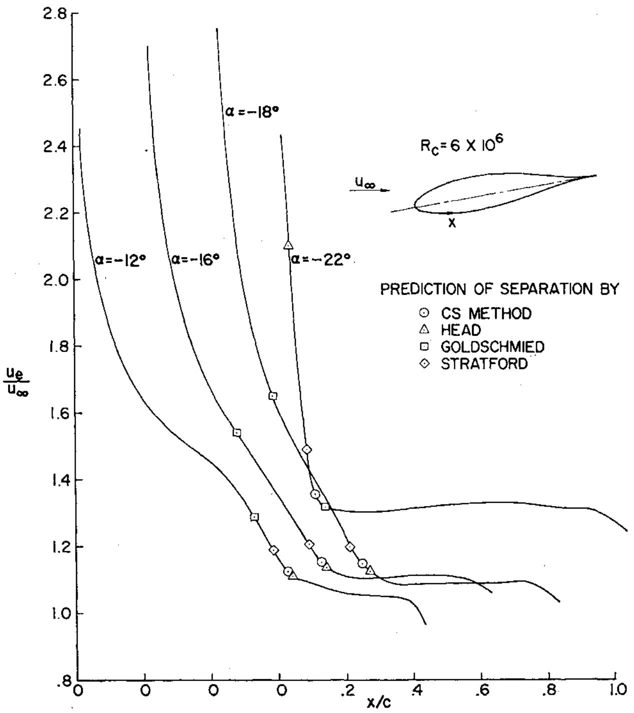

Fig. 12a Predicted separation points for the experimental pressure distribution on the NACA 65, 2-421 airfoil, negative angles of attack.

were obtained by the CS method by extrapolating the skin-friction values to zero. The agreement is excellent.

Calculations were also made for a flow past the sphere of 3-in. radius measured by Fage[19] for $R_D = 0.42 \times 10^6$ by using the CS method. The experimental separation point was not given, but was inferred from the experimental pressure distribution at an angular location of 140° from the stagnation point. The calculated value is 131°.

Summary

Based on the calculations shown in this paper, as well as many more (both unreported, and in Ref. 5) the following conclusions can be made on the accuracy of calculating the turbulent boundary-layer separation on two-dimensional and axisymmetric bodies:

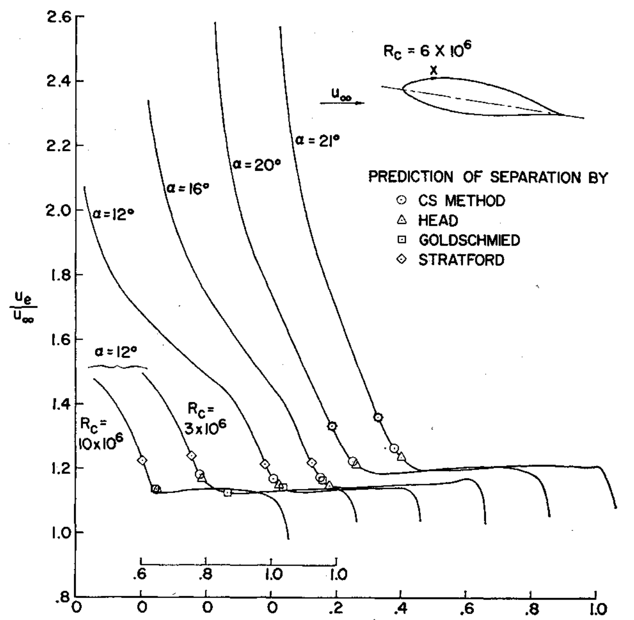

Fig. 12b Predicted separation points for the experimental pressure distribution on the NACA 65, 2-421 airfoil, positive angles of attack.

1) The location of turbulent boundary-layer separation on two-dimensional bodies can be calculated quite satisfactorily by the CS method, Head's method, and Stratford's method. Goldschmied's method is inconclusive. This is probably as a result of the very questionable assumption concerning the total pressure at the edge of the viscous sublayer. The results indicate that both boundary-layer methods predict the point of separation at approximately the same location. However, in some cases the CS predictions agree better with experiment than Head's predictions. Stratford's method is slightly conservative in its prediction but, is very convenient for calculation purposes.
2) The location of turbulent boundary-layer separation on axisymmetric bodies can be calculated quite accurately by the CS method. Head's, Stratford's, and Goldschmied's methods in their present form are not applicable to such flows.

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